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Erratum: Scaling dimensions of monopole operators in the $\mathbb{CP}^{N_b-1}$ theory in $2+1$ dimensions

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A missing factor of i was found in the computation of the mixed kernel $F_j^{q,B}(\omega)$. This results in minor changes in our formulas, but a major change in our conclusions. First, we correct the formulas, then we present the corrected conclusions.

Corrected formulas. The correct expression replacing (4.47) in the original article is

$$F_j^{q,B}(\omega) = \frac{16q\pi^2 i}{(2j+1)\sqrt{j(j+1)}} \sum_{j',j''=q}^{\infty} \left[\frac{E_{qj'}+E_{qj''}}{2E_{qj'}E_{qj''}(\omega^2+(E_{qj'}+E_{qj''})^2)}\right] \mathcal{I}_H(j,j',j'') \,. \tag{1}$$

Because $F_j^{q,B}(\omega)$ is pure imaginary the matrix of coefficients $\mathbf{M}_j^q(\omega)$ is not Hermitian, and (4.12) is modified to

$$\mathbf{M}_j^q(\omega) = \left(\begin{array}{c|ccc} D_j^q(\omega) & F_j^{q,B}(\omega) & F_j^{q,\tau}(\omega) & F_j^{q,E}(\omega) \\ \hline -F_j^{q,B*}(\omega) & K_j^{q,BB}(\omega) & K_j^{q,\tau B}(\omega) & K_j^{q,EB}(\omega) \\ -F_j^{q,\tau *}(\omega) & K_j^{q,\tau B*}(\omega) & K_j^{q,\tau\tau}(\omega) & K_j^{q,\tau E}(\omega) \\ -F_j^{q,E*}(\omega) & K_j^{q,EB*}(\omega) & K_j^{q,\tau E*}(\omega) & K_j^{q,EE}(\omega) \end{array}\right) . \tag{2}$$

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Consequently, the eigenvalues of the matrix are changed, (4.16) and (4.19) are changed to This matrix has eigenvalues:

$$
\begin{aligned}
\lambda_{\pm}^{q} &= \frac{(D_j^q(\omega) + K_j^{q,BB}(\omega)) \pm \sqrt{(D_j^q(\omega) - K_j^{q,BB}(\omega))^2 - 4|F_j^{q,B}(\omega)|^2}}{2}\,, \\
\lambda_{E}^{q} &= \frac{j(j+1)+\omega^2}{j(j+1)} K_j^{q,\tau\tau}\,,
\end{aligned}
\tag{3}
$$

and

$$
\delta\mathcal{F}_q = \frac{1}{2}\int \frac{d\omega}{2\pi}\left[\log\frac{D_0^q(\omega)}{D_0^0(\omega)} + \sum_{j=1}^{\infty}(2j+1)\log\frac{K_j^{q,\tau\tau}(\omega)\left[D_j^q(\omega)K_j^{q,BB}(\omega) + \left|F_j^{q,B}(\omega)\right|^2\right]}{D_j^0(\omega)K_j^{0,\tau\tau}(\omega)K_j^{0,BB}(\omega)}\right] \tag{4}
$$

respectively.

All formulas in the appendices are fixed by inserting an i in the appropriate places. The only nontrivial replacement is in (C.49), which correctly reads

$$
\begin{aligned}
L_j^q(\omega) = {} & \frac{8\mu_q^2}{(j+\frac{1}{2})^2+\omega^2} + \frac{12\mathcal{F}_q^\infty}{\pi}\frac{(j+\frac{1}{2})^2-\omega^2}{\left[(j+\frac{1}{2})^2+\omega^2\right]^{5/2}} \\
& - \frac{(q^2+4\mu_q^2(8\mu_q^2-1))(j+\frac{1}{2})^2 + 4(-q^2+\mu_q^2(8\mu_q^2-5))\omega^2}{2\left[(j+\frac{1}{2})^2+\omega^2\right]^3} \\
& + 144B_q\frac{3(j+\frac{1}{2})^4 - 24(j+\frac{1}{2})^2\omega^2 + 8\omega^4}{\left[(j+\frac{1}{2})^2+\omega^2\right]^{9/2}} \\
& + \frac{3\mathcal{F}_q^\infty}{2\pi}\frac{(25-48\mu_q^2)(j+\frac{1}{2})^4 + 3(64\mu_q^2-55)(j+\frac{1}{2})^2\omega^2 + 20\omega^4}{\left[(j+\frac{1}{2})^2+\omega^2\right]^{9/2}} \\
& + O\left(\frac{1}{\left[(j+\frac{1}{2})^2+\omega^2\right]^3}\right).
\end{aligned}
\tag{5}
$$

This change has important consequences on our final results. *All monopoles are stable, invalidating section 5.1.* The large q analysis supports the stability of monopoles; (5.9), (5.10) and figure 4 are replaced by

$$
\mathbf{M}_j^q(0) = \frac{\zeta\left(\frac{3}{2},\frac{1}{2}+\chi_0\right)}{8\pi\sqrt{2q}}\left(\begin{array}{c|ccc}
\frac{1}{2} & i\sqrt{j(j+1)}\,\chi_0 & 0 & 0 \\ \hline
i\sqrt{j(j+1)}\,\chi_0 & 2j(j+1)\,\chi_1 & 0 & 0 \\
0 & 0 & 4j(j+1)(\chi_0^2+\chi_1) & 0 \\
0 & 0 & 0 & 0
\end{array}\right), \tag{6}
$$

$$
\lambda_{\pm}^q \approx -\frac{0.055251 \pm 0.023717i}{\sqrt{q}}\,, \qquad \lambda_E^q \approx \frac{0.063044}{\sqrt{q}}\,. \tag{7}
$$

and figure 1.

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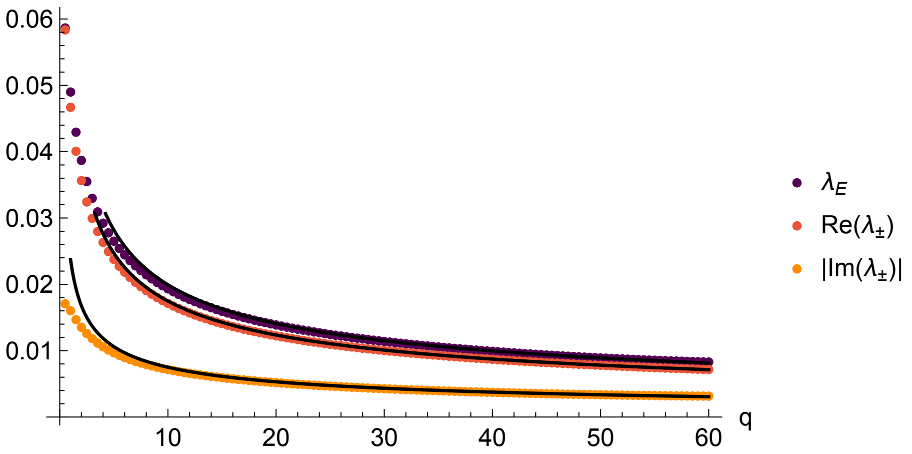

Figure 1. The numerical results for the three eigenvalues, λ_E^q, λ_+^q, and λ_-^q are plotted against the analytic large q value in black.

q	Δ_q	N_b for which $\Delta_q < 3$
0	0	$< \infty$
1/2	$0.1245922\, N_b + 0.3815 + O(N_b^{-1})$	≤ 21
1	$0.3110952\, N_b + 0.8745 + O(N_b^{-1})$	≤ 6
3/2	$0.5440693\, N_b + 1.4646 + O(N_b^{-1})$	≤ 2
2	$0.8157878\, N_b + 2.1388 + O(N_b^{-1})$	none
5/2	$1.1214167\, N_b + 2.8879 + O(N_b^{-1})$	none

Table 1. Results of the large N_b expansion of the monopole operator dimensions Δ_q obtained through calculating the ground state energy in the presence of $2q$ units of magnetic flux through S^2. In the last column of the table we listed our estimates for when the monopole operators are relevant.

Results and conclusions. Because these corrections make all saddle points stable, we are able to compute the dimensions of monopole for many values of q. The results are listed in table 1. In particular our result for $\Delta_{1/2}$ is different, and compares favorably with the numbers obtained by quantum Monte Carlo simulations of refs. [1–3] even for small N_b, as shown in the replacement of figure 1 of the original article in figure 2. From comparing the scaling dimensions collected in table 1 to 3, we can also estimate the upper bound on N_b below which the monopole operators are expected to be relevant; these bounds are also presented in table 1. There is inherently some uncertainty in these estimates, as they come from extrapolating the large N_b expansion to small values of N_b. Nevertheless, our relevance bounds come close to what ref. [3] found from numerics, as can be seen from table I in [3].

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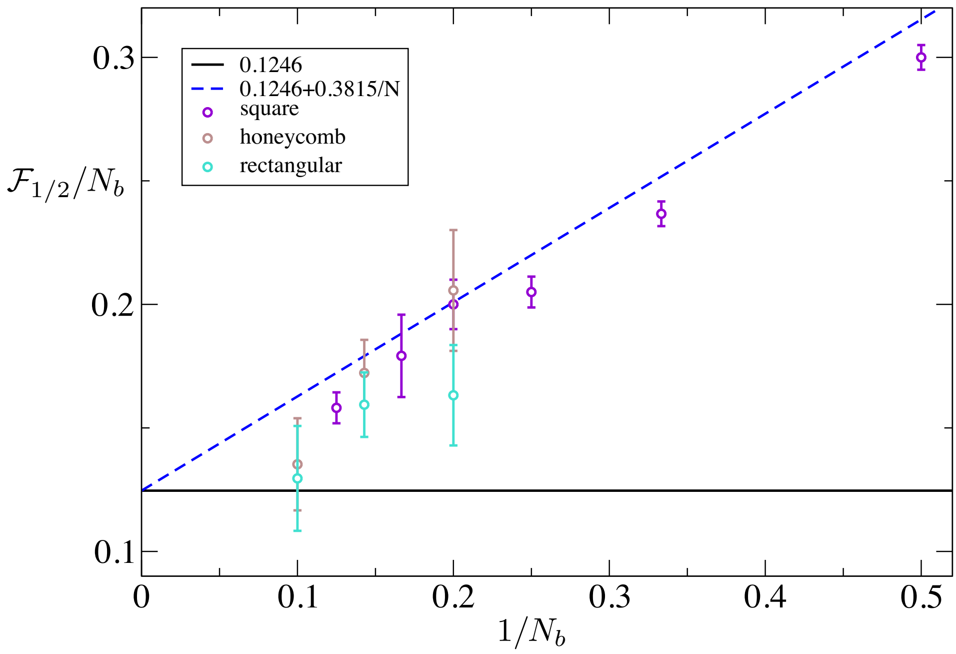

Figure 2. The scaling dimension of the $q = 1/2$ monopole operator, $\mathcal{F}_{1/2}$. The full line is the $N_b = \infty$ result (ref. [4]), and the dashed line is the leading $1/N_b$ correction computed in the present paper (see table 1). The quantum Monte Carlo results are for lattice antiferromagnets with global $SU(N_b)$ symmetry on the square (refs. [1, 2]), honeycomb (ref. [3]), and rectangular (ref. [3]) lattices.

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